Quantitative Risk Assessment

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Outline

Acknowledgment

My special thanks to:

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- Audun Josang for Subjective logic.

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- Chunming Rong for "Toilet Papers". :)

to make my job easier.

Risk Assessment Definition

- Risk assessment refers to the processes used to evaluate those probabilities and consequences, and also to the study of how to incorporate the resulting estimates into the decision-making process[1].

[1] Information Technology Risk Management In Enterprize Environments Book

Conventional Risk Assessment Methods

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- Based on *scoring*.
- Risk: risk is likelihood of a scenario times its consequence.
- No difference between high likelihood-low consequence events and low likelihood-high consequence events.
- Often interpreted as an expected value for risk, which contains too little information.

Conventional Risk Assessment Methods

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In a nutshell, the problems of scoring methods are three fold:

1. None of the scoring methods consider the issues about perception of risks and uncertainties.
2. The qualitative descriptions of likelihood, which is usually expressed by scores in the scoring methods, are understood and used very differently by different people.
3. The scoring schemes themselves add their own sources of error.

Conventional Risk Assessment Methods

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- However, the scoring methods are widely used to assess risk in different fields.
- If they are a failure, optimistically, they are a waste of time and money.
- In the worst case, the erroneous conclusions lead the organization down a more dangerous path that it would probably not have otherwise taken.

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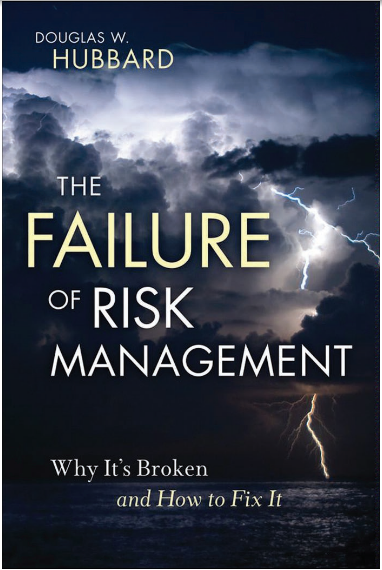
DOUGLAS W.
HUBBARD
THE
FAILURE
OF RISK
MANAGEMENT
Why It's Broken
and How to Fix It

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  - Likelihood of conversion of a source of danger (hazard) into actual delivery of loss, injury, or some form of damage.
- This notion involves both uncertainty and some kind of loss or damage that might be received.
- Measurement of uncertainty is done by assigning a set of probabilities(PDF) to a set of possibilities.

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$$R = \{\langle S_i, P_i, X_i \rangle\},\ i = 1, 2, ..., N \tag{1}$$

QRA Applications

- QRA is an effective method of exposing the risks of complex systems to events that could lead to catastrophic consequences.

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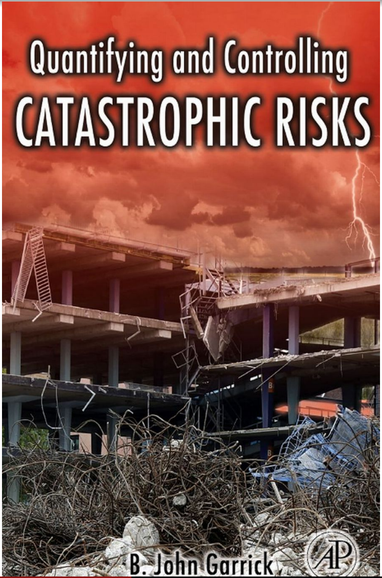
- QRA should be applied in cases where the consequences can be catastrophic and where there is great uncertainty about the risk scenarios and contributing factors.


Quantifying and Controlling
CATASTROPHIC RISKS
B. John Garrick
AP

Case Studies Reviewed

- Risk of a Catastrophic Hurricane in New Orleans, LA.
- Risk of Asteroids Impacting the Earth.
- Terrorist Attack on the National Electrical Grid.
- Abrupt Climate Change.

QRA Overview

- Step 1. Define the system being analyzed in terms of what constitutes normal operation.

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- Step 3. Develop "what can go wrong" scenarios to establish levels of damage and consequences.

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- Step 5. Assemble the scenarios according to damage levels, and cast the results into the appropriate risk curves.
- Step 6. Interpret the results to guide the risk management process.

Motivations

We decided to do a case study by this method on grading system at UoB in order to:

- Explain how the necessary computations in QRA are done in practice.

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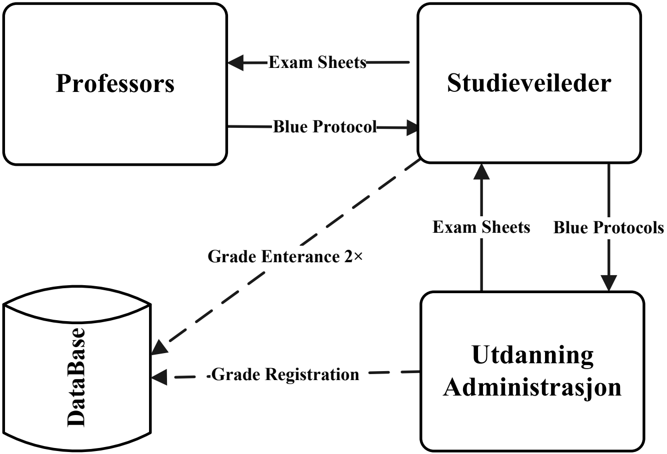
- Explain how the necessary computations in QRA are done in practice.
- Provide valuable information to personnel responsible for grade management at the university to give insight into where the largest risks are in the grade management system.

Step 1: Definition of the System During Normal Conditions



Step 2: Identification and Characterization of System Hazards

We can divide hazards into 8 following classes[3]:

- Disasters
- Abuses
- Technical Errors
- Active Attacks
- Human Errors
- Passive Attacks
- Physical Attacks
- Maintenance Failures

[3] ISO/IEC TR 13335 Annex D

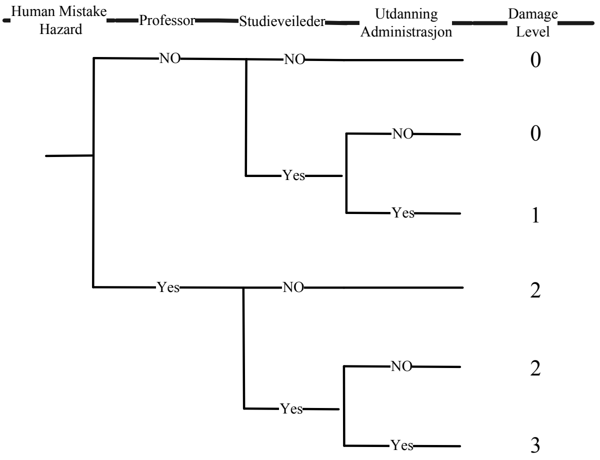
Scope of our project

- Studying the impact of honest human error (mistakes) as a source of danger on the grading system at UoB database.

Step 3: Structuring of the Risk Scenarios and Consequences

- **Damage Level 0:** No thing happen.
- **Damage Level 1:** mistake in the digital data.
- **Damage Level 2:** mistake in the paper data (It will propagate to digital data).
- **Damage Level 3:** mistake in the both paper and digital data.

Step 3: Structuring of the Risk Scenarios and Consequences



Step 3: Structuring of the Risk Scenarios and Consequences

- Scenario 1: Professors make no mistake, but some mistakes happen in Studentveileder and the same mistake takes place in Utdanning Administrasjon, this scenario causes only some mistake in the digital data and therefore results in damage level 1.

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- Scenario 1: Professors make no mistake, but some mistakes happen in Studentveileder and the same mistake takes place in Utdanning Administrasjon, this scenario causes only some mistake in the digital data and therefore results in damage level 1.
- Scenario 2: Professors make some mistakes but no mistakes happen in Studentveileder and naturally there will be no mistake to be detected at Utdanning Administrasjon . This scenario causes only some mistake in the paper data and therefore results in damage level 2.

Step 3: Structuring of the Risk Scenarios and Consequences

- Scenario 3: Professors make some mistakes, some other mistakes also happen in Studentveileder, but the latter mistake is detected by the Utdanning Administrasjon . This scenario also causes only some mistake in the paper data and therefore results in damage level 2.

Step 3: Structuring of the Risk Scenarios and Consequences

- Scenario 3: Professors make some mistakes, some other mistakes also happen in Studentveileder, but the latter mistake is detected by the Utdanning Administrasjon . This scenario also causes only some mistake in the paper data and therefore results in damage level 2.
- Scenario 4: Professors make some mistakes, some other mistakes also happen in Studentveileder, and the latter mistake is not detected by the Utdanning Administrasjon . This scenario causes some mistakes in both paper and digital data and therefore results in damage level 3.

Step4: Quantification of the Likelihood of the Scenarios

1. Identification of Elemental Components.

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  1. Professors (P)
  2. Studentveileder(S)
  3. Utdanning Administrasjon(U)

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  2. A range or an interval e.g. [a,b].

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$$P_0(E) = \int_0^1 P_0(\lambda)P(E|\lambda)d\lambda$$

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- $S_2 = p.\overline{s}.\overline{u}$
- $S_3 = p.s.\overline{u}$
- $S_4 = p.s.u$

e.g. $\overline{p} = 1 - p$

Finding a Relationship between each Scenario and Elemental Components.

Now, the likelihood of each scenario should be presented as a function of the likelihood of the elemental events.

$$
\begin{aligned}
P(S_1) &= P(\overline{p}.s.u) = P(s.u - p.s.u) \\
P(S_2) &= P(p.\overline{s}.\overline{u}) = P(p - p.u - p.s + p.s.u) \\
P(S_3) &= P(p.s.\overline{u}) = P(p.s - p.s.u) \\
P(S_4) &= P(p.s.u)
\end{aligned}
$$

5. Finding the Likelihood (PDF) of Scenarios

- In order to use the previous equations to calculate the probability density of each scenario, we should first know how to calculate the probability density of product and summation of two variables.

Remark1: Product of two random variables

Rohatgi's well-known result for determining the distribution of the product of two random variables is straightforward to derive, but difficult to implement.

- Let X and Y be continuous random variables with joint PDF $P_{X,Y}(x, y)$. The PDF of $V = XY$ is

$$P_V(v) = \int_{-\infty}^{\infty} P_{X,Y}(x, \frac{v}{x}).\frac{1}{|x|}dx \tag{2}$$

Remark2: Summation of two random variables

- Let X and Y be two continuous random variables with density functions $P_X(x)$ and $P_Y(y)$, respectively.
- Then the sum $Z = X + Y$ is a random variable with density function $P_V(v)$:

$$P_V(v) = \int_{-\infty}^{\infty} P_Y(v - x).P_X(x)dx \qquad (3)$$

5. Finding the Likelihood (PDF) of Scenarios

- By using the aforementioned Remarks, we can calculate the probability density curve for each scenario.

PDF of Scenario 1 ($P(S_1)$)

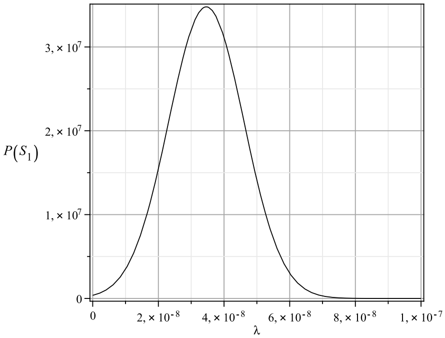

Figure: S_1 PDF

PDF of Scenario 2 ($P(S_2)$)

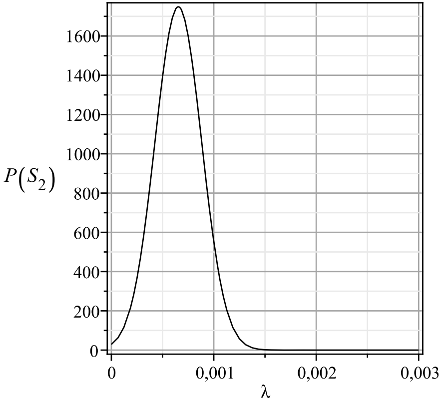

Figure: S_2 PDF

PDF of Scenario 3 ($P(S_3)$)

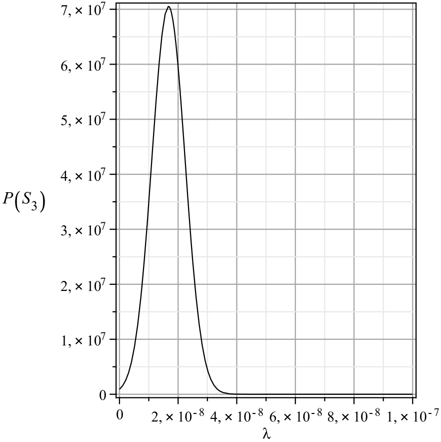

Figure: S_3 PDF

PDF of Scenario 4 ($P(S_4)$)

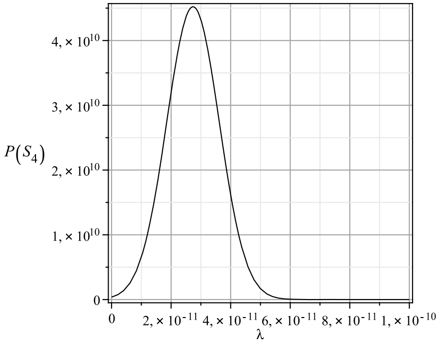

Figure: S_4 PDF

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- Then, the scenarios are arranged in order of increasing damage levels.

Step 5. Assemble the scenarios and calculate of risk curves

Scenarios	Likelihood	Damage Level
S_1	$P(S_1)$	1
$S_2 + S_3$	$P(S_2 + S_3)$	2
S_4	$P(S_4)$	3

Table: Scenario List

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- The most common form is the classical risk curve, also known as the frequency-of-exceedance curve, or the complementary-cumulative-distribution-function.

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- To find this curve, we can first add a fourth column in which we write the cumulative probability, adding from the bottom.

Calculation of risk curves

Scenarios	Likelihood	Damage Level	Cumulative Probability
S1	$P(S_1)$	1	$\Phi_1 = \Phi_2 + S_1$
S2+S3	$P(S_2 + S_3)$	2	$\Phi_2 = \Phi_3 + S_2 + S_3$
S4	$P(S_4)$	3	$\Phi_3 = S_4$

Table: Scenario List with Cumulative Probability

Calculation of risk curves

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- $\Phi_i = \Phi_{i+1} + \phi_i$, Φ_i=frequency of damage x_i or greater.
- The PDF of each Φ_i denoted as $\prod_i(\phi_i)$:

$$\prod_i(\Phi_i) = \int_0^{\Phi_i} \prod_{i+1}(\Phi_{i+1}) p_i(\Phi_i - \Phi_{i+1}) d\Phi_{i+1} \qquad (4)$$

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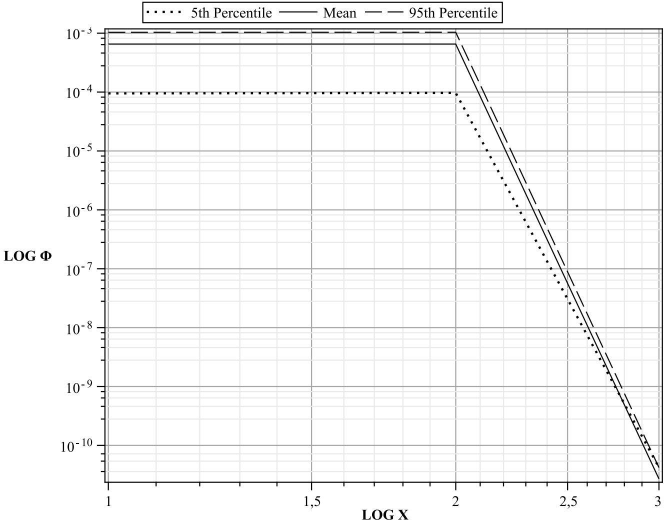
Calculation of risk curves



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- The uncertainty analysis indicates that there is 90% confidence that the actual frequency lies within 2.7E-4 and 1.0E-3 for any damage level in 5th and 95th percentile respectively.

Step6: Interpretation of the results

Human Mistake Rate (Mistake pre Grade)			
Damage Level	**5th Percentile**	**95th Percentile**	**Mean**
Any Damage	2.7E-4	1.0E-3	6.5E-4
1	1.6E-8	5.3E-8	3.4E-8
2	2.8E-4	1.0E-3	6.5E-4
3	1.3E-11	4.2E-11	2.7E-11

Table: Selected parameters of uncertainty distribution for each level of damage

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  - We are 90% confident that the frequency of this consequence is between 2.8E-4 and 1.0E-3 or once in 3571 and 1000 grades respectively.
  - This consequence is mainly influenced by mistakes made by professors while they enter the grades wrongly in the blue protocol (scenario 2).

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- However, other consequences which include mistakes in digital data (damage level 1 and 3) are quite unlikely.
  - The frequencies of having damage level 1 and 3 on average are 3.4E-8 and 2.7E-11 respectively.
  - That is approximately once in 29 million and 37 billion grades.

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- Applying on a more sophisticated system.